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Coherent states and Berezin transforms attached to Landau levels

ABSTRACT. In general, coherent states $(|x\rangle)_{x\in X}$ are a specific overcomplete family of normalized vectors in the Hilbert space $\mathcal{H}$ of the problem that describes the quantum phenomena and solves the identity of $\mathcal{H}$ as

$$1_{\mathcal{H}} = \int_X |x\rangle \langle x| \, d\mu(x).$$

These states have long been known for the harmonic oscillator and their properties have frequently been taken as models for defining this notion for other models. We review the definition and properties of coherent states with examples. We construct coherent states attached to Landau levels (discrete energies of a uniform magnetic field) on three known examples of Kähler manifolds X: the Poincaré disk $\mathbb{D}$, the Euclidean plane $\mathbb{C}$ and the Riemann sphere $C\mathbb{P}^1$. After defining their corresponding integral transforms, we obtain characterization theorems for spaces of bound states of the particle. Generalization to $\mathbb{C}^n$ and to the complex unit ball $\mathbb{B}^n$ and $C\mathbb{P}^n$ are also discussed. In these cases, we apply a coherent states quantization method to recover the corresponding Berezin transforms and we give formulae representing these transforms as functions of Laplace-Beltrami operators.