talk

CLIFFORD ALGEBRA IMPLEMENTATIONS IN MAXIMA

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Maxima is the open source descendant of the first ever computer algebra system and features a rich functionality from a large number of shared packages. While written in Lisp, *Maxima* has its own programming language, based on Lisp. The Maxima language is based on the ideas of functional programming, which is particularly well suited for formal transformations of mathematical expressions. The packages *clifford* and *cliffordan* authored by the presenter, implement Clifford algebras $C\ell_{p,q,r}$ of arbitrary signatures and order. The *clifford* package defines multiple rules for pre- and post-simplification of Clifford products, outer products, scalar products, inverses and powers of Clifford vectors [1]. Using this functionality any combination of products can be put into a canonical representation, for example in the quaternion algebra $C\ell_{0,2}$:

```
mtable1([1, e[1],e[2], e[1] . e[2]]);
```

$$\begin{pmatrix} 1 & e_1 & e_2 & e_1.e_2 \\ e_1 & -1 & e_1.e_2 & -e_2 \\ e_2 & -e_1.e_2 & -1 & e_1 \\ e_1.e_2 & e_2 & -e_1 & -1 \end{pmatrix}$$

$$block(declare([a,b,c,d],scalar),cc : a+b*e[1]+c*e[2]+d*e[1].e[2],dd : cinv(cc))$$

$$\frac{a-e_1\,b-e_2\,c-(e_1.e_2)\,d}{a^2+b^2+c^2+d^2}$$

The inner product is represented by the operator symbol "|" and the outer (exterior, or wedge) product by the operator symbol "&". For example the sum of the inner and outer products of two elements immediately simplifies into the full Clifford product:

```
a | b + a & b;
```

$$a \cdot b$$

or the Jacobi identity automatically holds for the even-grade multivectors:

```
a & b & c + b & c & a + c & a & b;
```

$$0$$

The presentation will demonstrate applications of *clifford* and *cliffordan* in linear algebra and calculus.

REFERENCES

[1] D. Prodanov and V. T. Toth. Sparse representations of Clifford and tensor algebras in Maxima. *arXiv:1604.06967*, 2016.

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